

Goniometria

Valori delle funzioni goniometriche					
gradi	radiani	Seno	Coseno	Tangente	Cotangente
0	0	0	1	0	Non esiste
9	$\pi/20$	$\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$	$\frac{4-\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$	$\frac{\sqrt{5}-1}{4-\sqrt{10+2\sqrt{5}}}$
15	$\pi/12$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\pi/10$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
22,5	$\pi/8$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
30	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
36	$\pi/5$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
45	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
54	$3\pi/10$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
60	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
67,5	$3\pi/8$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
72	$2\pi/5$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
75	$5\pi/12$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
81	$9\pi/20$	$\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4-\sqrt{10+2\sqrt{5}}}$	$\frac{4-\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
90	$\pi/2$	1	0	Non esiste	0

Valori delle funzioni seno e coseno per angoli multipli di 3° fino a 45°			
gradi	radianti	Seno	Coseno
0	0	0	1
3	$\pi/60$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}-1)-2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}}{16}$	$\frac{2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}+\sqrt{2}(\sqrt{3}-1)(\sqrt{5}-1)}{16}$
6	$\pi/30$	$\frac{\sqrt{6}\sqrt{5-\sqrt{5}}-(\sqrt{5}+1)}{8}$	$\frac{\sqrt{2}\sqrt{5-\sqrt{5}}-\sqrt{3}(\sqrt{5}+1)}{8}$
9	$\pi/20$	$\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$	$\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$
12	$\pi/15$	$\frac{\sqrt{2}\sqrt{5+\sqrt{5}}-\sqrt{3}(\sqrt{5}-1)}{8}$	$\frac{\sqrt{6}\sqrt{5+\sqrt{5}}+\sqrt{5}-1}{8}$
15	$\pi/12$	$\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4} = \frac{\sqrt{2+\sqrt{3}}}{2}$
18	$\pi/10$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$
21	$\pi/8$	$\frac{2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-\sqrt{2}(\sqrt{3}-1)(\sqrt{5}+1)}{16}$	$\frac{2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}+\sqrt{2}(\sqrt{3}+1)(\sqrt{5}+1)}{16}$
24	$2\pi/15$	$\frac{\sqrt{3}(\sqrt{5}+1)-\sqrt{2}\sqrt{5-\sqrt{5}}}{8}$	$\frac{\sqrt{5}+1+\sqrt{6}\sqrt{5-\sqrt{5}}}{8}$
27	$3\pi/20$	$\frac{2\sqrt{5+\sqrt{5}}-\sqrt{2}(\sqrt{5}-1)}{8}$	$\frac{2\sqrt{5+\sqrt{5}}+\sqrt{2}(\sqrt{5}-1)}{8}$
30	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
33	$11\pi/60$	$\frac{2(\sqrt{3}-1)(\sqrt{5+\sqrt{5}})+\sqrt{2}(\sqrt{3}+1)(\sqrt{5}-1)}{16}$	$\frac{2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-\sqrt{2}(\sqrt{3}-1)(\sqrt{5}-1)}{16}$
36	$\pi/5$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$
39	$13\pi/60$	$\frac{\sqrt{2}(\sqrt{3}+1)(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}}{16}$	$\frac{\sqrt{2}(\sqrt{3}-1)(\sqrt{5}+1)+2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}}{16}$
42	$7\pi/30$	$\frac{\sqrt{6}\sqrt{5+\sqrt{5}}-(\sqrt{5}-1)}{8}$	$\frac{\sqrt{2}\sqrt{5+\sqrt{5}}+\sqrt{3}(\sqrt{5}-1)}{8}$
45	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

Relazioni fra le funzioni goniometriche di uno stesso angolo

	$\text{sen}\alpha$	$\text{cos}\alpha$	$\text{tg}\alpha$	$\text{ctg}\alpha$
$\text{sen}\alpha$	$\text{sen}\alpha$	$\pm\sqrt{1-\text{sen}^2\alpha}$	$\frac{\pm\text{sen}\alpha}{\sqrt{1-\text{sen}^2\alpha}}$	$\frac{\pm\sqrt{1-\text{sen}^2\alpha}}{\text{sen}\alpha}$
$\text{cos}\alpha$	$\pm\sqrt{1-\text{cos}^2\alpha}$	$\text{cos}\alpha$	$\frac{\pm\sqrt{1-\text{cos}^2\alpha}}{\text{cos}\alpha}$	$\frac{\pm\text{cos}\alpha}{\sqrt{1-\text{cos}^2\alpha}}$
$\text{tg}\alpha$	$\frac{\pm\text{tg}\alpha}{\sqrt{1+\text{tg}^2\alpha}}$	$\frac{\pm 1}{\sqrt{1+\text{tg}^2\alpha}}$	$\text{tg}\alpha$	$\frac{1}{\text{tg}\alpha}$
$\text{ctg}\alpha$	$\frac{\pm 1}{\sqrt{1+\text{ctg}^2\alpha}}$	$\frac{\pm\text{ctg}\alpha}{\sqrt{1+\text{ctg}^2\alpha}}$	$\frac{1}{\text{ctg}\alpha}$	$\text{ctg}\alpha$

Relazioni goniometriche tra angoli associati

Angoli opposti	Angoli supplementari	Angoli che differiscono di un angolo piatto
$\text{sen}(-\alpha) = -\text{sen}\alpha$ $\text{cos}(-\alpha) = \text{cos}\alpha$ $\text{tg}(-\alpha) = -\text{tg}\alpha$	$\text{sen}(\pi - \alpha) = \text{sen}\alpha$ $\text{cos}(\pi - \alpha) = -\text{cos}\alpha$ $\text{tg}(\pi - \alpha) = -\text{tg}\alpha$	$\text{sen}(\pi + \alpha) = -\text{sen}\alpha$ $\text{cos}(\pi - \alpha) = -\text{cos}\alpha$ $\text{tg}(\pi - \alpha) = +\text{tg}\alpha$

Angoli esplementari	Angoli complementari	Angoli che differiscono di un angolo retto
$\text{sen}(2\pi - \alpha) = -\text{sen}\alpha$ $\text{cos}(2\pi - \alpha) = \text{cos}\alpha$ $\text{tg}(2\pi - \alpha) = -\text{tg}\alpha$	$\text{sen}\left(\frac{\pi}{2} - \alpha\right) = \text{cos}\alpha$ $\text{cos}\left(\frac{\pi}{2} - \alpha\right) = \text{sen}\alpha$ $\text{tg}\left(\frac{\pi}{2} - \alpha\right) = \text{cotg}\alpha$	$\text{sen}\left(\frac{\pi}{2} + \alpha\right) = \text{cos}\alpha$ $\text{cos}\left(\frac{\pi}{2} + \alpha\right) = -\text{sen}\alpha$ $\text{tg}\left(\frac{\pi}{2} + \alpha\right) = -\text{cotg}\alpha$

Formule goniometriche	
Formule di addizione	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\operatorname{tg}\beta}$
Formule di sottrazione	$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha\operatorname{tg}\beta}$
Formule di duplicazione	$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$ $= 2\cos^2\alpha - 1$ $\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$
Formule di triplicazione	$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ $\operatorname{tg} 3\alpha = \frac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1 - 3\operatorname{tg}^2\alpha}$
Formule di bisezione	$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos\alpha}{2}}$ $\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos\alpha}{2}}$ $\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{\sin\alpha}{1 + \cos\alpha} = \frac{1 - \cos\alpha}{\sin\alpha}$
Formule parametriche	$\sin\alpha = \frac{2t}{1+t^2}$ $\cos\alpha = \frac{1-t^2}{1+t^2}$ $\operatorname{tg}\alpha = \frac{2t}{1-t^2}$ $t = \operatorname{tg} \frac{\alpha}{2}$
Formule di linearizzazione	$\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$ $\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$ $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$

Formule di prostaferesi	$\operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen} \frac{p+q}{2} \cos \frac{p-q}{2}$ $\operatorname{sen} p - \operatorname{sen} q = 2 \cos \frac{p+q}{2} \operatorname{sen} \frac{p-q}{2}$ $\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$ $\cos p - \cos q = -2 \operatorname{sen} \frac{p+q}{2} \operatorname{sen} \frac{p-q}{2}$
Formule di Werner	$\operatorname{sen} \alpha \operatorname{sen} \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$ $\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$ $\operatorname{sen} \alpha \cos \beta = \frac{1}{2} [\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta)]$